

# MAS212 Linear Algebra I 2007-2008

## Coursework 1

Please hand in your solutions of the **starred** exercises by **1pm on Friday 5 October** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

**Exercise\* 1.** Let  $p$  be a quadratic polynomial  $p(x) = ax^2 + bx + c$ . Given that  $p(-1) = 6$ ,  $p(1) = 0$ , and  $p(2) = 3$ , write down a system of 3 linear equations satisfied by the unknown coefficients  $a$ ,  $b$ , and  $c$ .

**Exercise 2.** Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{aligned} \text{(a)} \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 0 & 1 & 5 \end{pmatrix} \quad \text{(b)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad \text{(c)} \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{(d)} \begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 5 \end{pmatrix} \\ \text{(e)} \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{pmatrix} \quad \text{(f)} \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{(g)} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

**Exercise 3.** In each of the following, the augmented matrix is in reduced row echelon form. In each case, identify the leading variables and the free variables, and determine the solution set of the corresponding linear system.

$$\begin{aligned} \text{(a)} \left( \begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right) \quad \text{(b)} \left( \begin{array}{cccc|c} 1 & 2 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 & 4 \end{array} \right) \quad \text{(c)} \left( \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \quad \text{(d)} \left( \begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{aligned}$$

**Exercise\* 4.** Consider the linear system

$$\begin{aligned} x_1 &- x_2 + 2x_3 = 3 \\ -x_1 &+ 2x_2 + x_3 = -5 \\ x_1 &+ x_2 + 9x_3 = 0. \end{aligned}$$

- Write down the augmented matrix of the system.
- Use Gaussian elimination to bring the augmented matrix to row echelon form and, as in the lectures, indicate which elementary row operations are used at each step.
- Identify the leading and the free variables, and write down the solution set of the system.

**Exercise\* 5.** Consider the linear system

$$\begin{aligned} x_1 &+ 2x_2 + x_3 + 2x_4 = 3 \\ 2x_1 &+ 4x_2 + 3x_3 + 7x_4 = 7 \\ -x_1 &- 2x_2 + x_4 = -2. \end{aligned}$$

- Write down the augmented matrix of the system.
- Use Gauss-Jordan reduction to bring the augmented matrix to reduced row echelon form and, as in the lectures, indicate which elementary row operations are used at each step.
- Identify the leading and the free variables, and write down the solution set of the system.

**Exercise 6.** Solve the  $3 \times 3$  linear system in the unknowns  $a$ ,  $b$ , and  $c$  obtained in Exercise 1 using Gaussian elimination, and, hence, determine the polynomial  $p$ .